Abstract – In this paper we analyze the performance in terms of Bit Error Rate (BER) and Frame Error Rate (FER) of a multi-binary turbo coded (MBTC) system over Rice flat fading multipaths channels, for different unfading and fading power ratio values. The MBTCs proposed are made up by the parallel concatenation of two identical rate 2/3 recursive systematic convolutional RSC duo-binary codes. Data block with 188 bytes (1504 bits) length, a QPSK modulation, MaxLogMAP algorithm, 15 iterations and a stop criterion of iterations were used employed.

Keywords: multi binary turbo code, Rice flat fading channel, MaxLogMAP algorithm

I. INTRODUCTION

More than ten years after their appearance, [1], turbo codes (TCs) constitute a solution for the improvement of the BER performances in any communication system. The interleaver presence in the turbo coding weakens their performance prediction for a certain configuration of the communication system. Thus, the most real way to evaluate this performance remains the Monte Carlo method [2]. This paper shows the performance of MBTCs in the Rice flat fading channel. The structure of this paper is the following. Paragraph I presents the models used in the simulations for the Rice channel and the turbo-coded system. In paragraph II a succinct presentation of the MBTC is made. Paragraph III provides the simulation results. In the last paragraph some concluding remarks are given.

The Rice flat fading models [3] the radio communications channels, for which the received signal has a direct component (assimilated to a non-fading channel) and a fading component (assimilated to a Rayleigh channel – Fig.1). The power balance between the two components give a measure of the Rice fading, quantified in the following with the coefficient:

\[ K = \frac{\text{unbalanced power component}}{\text{total power}} \]  

The variation of the BER function of the \( K \) coefficient, for two values of the signal to noise ratio (SNR), in the non-coding transmission case is presented in Fig.2. We can notice that the effect of the fading is essential even for small proportions of the fading component, given by \( 1-K \) difference. This effect is for small SNR even more obvious values. Using the notation from Fig.1, the input-output relation is the following:

\[ y_k = y_{fk} + y_{uk} = x_k \cdot \alpha_k + w_k, \]
where \( r_k \) is a Rice random variable, composed by a Rayleigh random variable, \( \alpha_k \), on \( \sqrt{1-K} \) proportion, and a \( \sqrt{K} \) constant. Thus, it results that \( r_k^2 = 1 \). Considering for \( x_k \) a unitary power too, the SNR value can be:

\[
\frac{E_b}{N_o} = \frac{1}{2} \cdot \frac{2 \cdot E_b}{N_o} = \frac{1}{2} \frac{r_k^2 \cdot x_k^2}{w_k^2} = \frac{1}{2} \frac{1}{w_k^2}.
\]

so, for simulations, the noise power value is:

\[
\frac{w_k^2}{k} = \frac{1}{2 \cdot 10^{2 \cdot SNR / 10}} \tag{4}
\]

In the case of applying the turbo coding, the model of the system has the configuration from Fig.3. The BER computation is made by comparing the sequences from the input on turbo-coder and after the turbo decoder. The relation between \( y \) and \( x \) is given by relation (2).

II. MULTI BINARY TURBO CODES

The novelty brought by MBTC consists in the usage of the multi input coders as component codes [4]. The general scheme of a multi input coder is presented in Fig.4. Using the notations:

\[
S_t = [s^t_m \ldots s^t_2 s^t_1]^T = \text{the vector „state” of the coder at the } t \text{ moment},
\]

\[
U_t=[u^t_r u^t_{r-1} \ldots u^t_1]^T = \text{the „input” vector at the } t \text{ moment},
\]

\[
H = \begin{bmatrix}
  h_{r+1,m} & h_{r,m} & \ldots & h_{1,m} & h_{0,m} \\
  \vdots & \vdots & \ddots & \vdots & \vdots \\
  h_{r+1,2} & h_{r,2} & \ldots & h_{1,2} & h_{0,2} \\
  h_{r+1,1} & h_{r,1} & \ldots & h_{1,1} & h_{0,1}
\end{bmatrix} = H_{m1} = \text{the complete generating matrix},
\]

\[
H_0 = \begin{bmatrix}
  h_{r,m} & \ldots & h_{1,m} \\
  \vdots & \ddots & \vdots \\
  h_{r,2} & \ldots & h_{1,2} \\
  h_{r,1} & \ldots & h_{1,1}
\end{bmatrix} = H_{m1} = \text{the limited generating matrix},
\]

\[
H_R = [h_{0,m} \ldots h_{0,2} h_{0,1}]^T = \text{the “feedback” vector},
\]

\[
H_F = [h_{r+1,m} \ldots h_{r+1,2} h_{r+1,1}]^T = \text{the output vector},
\]

where the \( T \) exponent means the transposing operation, the main equation of the circuit from Fig.4 is:

\[
(S_{t+1})_{m1} = (H_0)_{m1} \cdot (U_t)_{r+1} + (H_R)_{m1} \cdot (S_t)_{m1}, \tag{5}
\]

where \( T \) matrix is:

\[
T = \begin{bmatrix}
  0 & 1 & 0 & \ldots & 0 & 0 \\
  0 & 0 & 1 & \ldots & 0 & 0 \\
  0 & 0 & 0 & \ldots & 0 & 1 \\
  \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
  0 & 0 & 0 & \ldots & 0 & 1 \\
  h_{0,m} & h_{0,m-1} & h_{0,m-2} & \ldots & h_{0,2} & h_{0,1}
\end{bmatrix}.
\]
The MBTCs present a series of advantages as to the uni-binary TCs [5], as: – a better convergence of the iterative decoding process; – a superior code minimum distance; – they are less sensitive to the negative effects of the puncturing; – the decrease of the delay due to the interleaving; – the robustness of the decoder; – the possibility of the direct utility of the high-order modulation [6]. In this paper we have investigated the way in which these advantages of the MBTCs, demonstrated in case of using them in non-fading channels, are also valid in the case of the Rice flat-fading channel.

III. EXPERIMENTAL RESULTS

The simulations presented in this paragraph were made with TCs, having their parameters defined in Table 1. The chosen parameters, were mainly the ones used in [4]. The trellis closing is the exception which wasn’t made circular. The simulations of the results presented in BER and FER diagrams, on Fig. 5 b) and c), show almost identical performance. It is the case of the curves on the extreme left from the FER diagram, curves which correspond to the non-fading case ($K=1$).

In both diagrams, the curves which correspond to the 3 memory MBTC, traced with continuous line, while the curves traced with a broken line correspond to the 4 memory MBTC. In every case we have drawn five curves corresponding to the five values of $K$ coefficient: 0, 0.25, 0.5, 0.75 and 1. The case $K=0$ (curves from extreme right) corresponds to a non-fading channel (AWGN).

Table 1 Parameters of the simulated turbo coded

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Used variant</th>
</tr>
</thead>
<tbody>
<tr>
<td>The turbo code configuration</td>
<td>Parallel</td>
</tr>
<tr>
<td>The component code</td>
<td>RSC code with: 3 memory ($H=[6\ 7\ 1\ 5]$) and 4 memory ($H=[11\ 11\ 11\ 12]$)</td>
</tr>
<tr>
<td>Turbo coding rate</td>
<td>1/2</td>
</tr>
<tr>
<td>Punctured</td>
<td>no</td>
</tr>
<tr>
<td>Modulation</td>
<td>QPSK</td>
</tr>
<tr>
<td>Channel</td>
<td>Rice flat fading with $K$ variable</td>
</tr>
<tr>
<td>Interleaving</td>
<td>Intersymbol and intrasymbol interleaving defined in [4]</td>
</tr>
<tr>
<td>Interleaver</td>
<td>Adapted to the component code, defined in [4]</td>
</tr>
<tr>
<td>Data block length</td>
<td>188bytes = 2 x 752 bits</td>
</tr>
<tr>
<td>Decoding algorithm</td>
<td>MaxLogMAP with decoding per symbol and with the ponderating coefficient of the extrinsic information equal to 0.75</td>
</tr>
<tr>
<td>Iteration number</td>
<td>15 iterations with a stop criterion iteration based on APP (A Posteriori Probability) distribution.</td>
</tr>
<tr>
<td>Simulated block number</td>
<td>Inverse proportionally with the logarithm of the BER.</td>
</tr>
</tbody>
</table>

Figura 5 The performance of MBTCs of the power balance $K$

To compare, in the diagram from Fig. 5 a) were drawn BER curves obtained from the simulation of the uncoded Rice channel, for the same 5 values of the $K$ parameters.

IV. CONCLUSIONS

As was known [7], [8], [9], the turbo codes offer a gain of tens decibels in the case of the flat fading channel. For example for $K=1$ (unfading case) this gain is about 9dB for $BER=10^{-5}$. For the same BER, for $K=0.75$ the coding gain is 35dB, for $K=0.5$ is 40dB, while for $K=0$ (Rayleigh channel) it cannot even be estimated because, in the uncoded case, it cannot hit a $BER=10^{-5}$.

Another remarkable fact is the following. If in the uncoded case, the presence of the continuous component cannot be seen in BER, unless it is the dominant component (for values over than 0.8 of the $K$ parameter), in the case of
applying the turbo-coding, the BER is visibly improved with the growing of the $K$ parameter, starting even with values close to 0.

Making a comparison between the performance of the uni-binary TCs from the Rice channel, given in [7] and [8], and the MBTCs, the first thing we observe (in diagram c) from Fig.5 is a better MBTCs convergence. Especially for the 4 memory code, the curves are very abrupt even at low values of the BER. In addition to this, we have the advantages referring the delay caused by the interleaving (twice as small for the MBTCs) and to the possibility of directly using the high-order modulations.

REFERENCES


[4] Catherine Douillard, Claude Berrou, “Turbo Codes With Rate-$m/(m + 1)$ Constituent Convolutional Codes”


